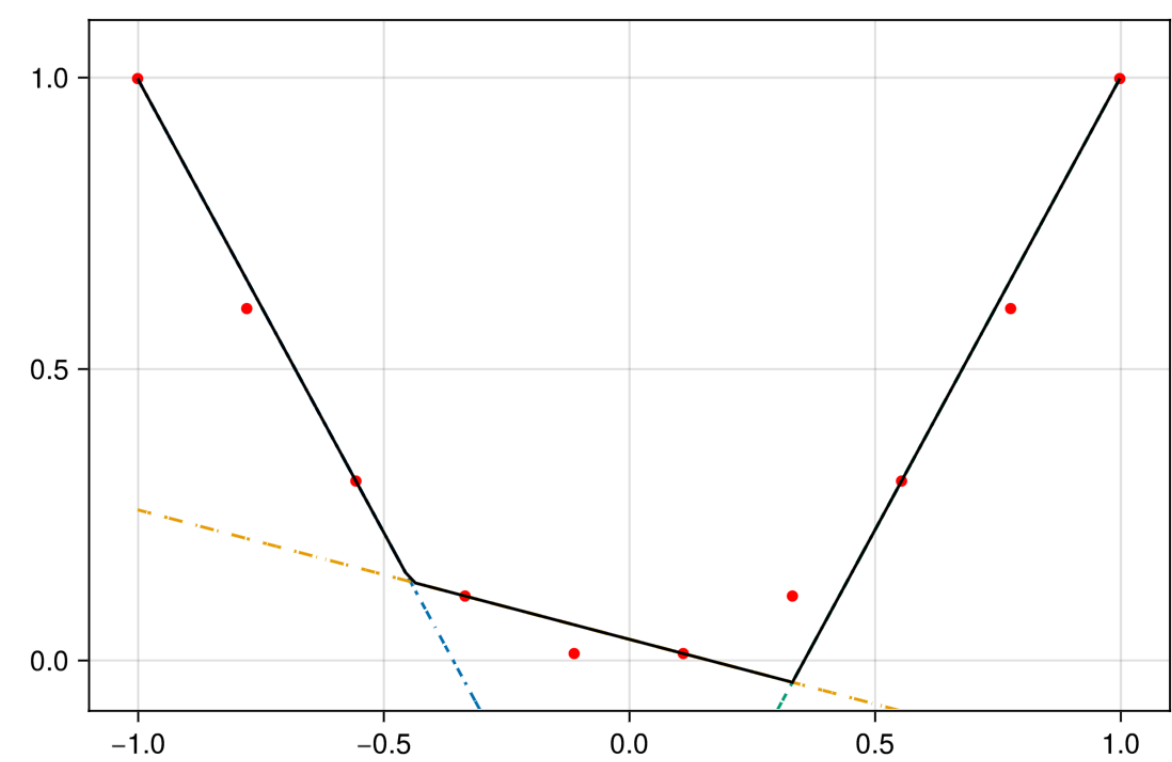


Present

# PiecewiseAffineApprox.jl

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# Agenda

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- About Us
- Motivation
- Demo
- Energy Applications
- Future Improvements

# About Us

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- SINTEF is one of Europe's largest independent research organisations (~2200 employees).
- Department of Sustainable Energy Technology, Optimization group
- MILP optimization (mostly modelling) for different sectors:
  - Supply chain optimization
  - Energy systems
  - Health care
  - Sustainability
- Moving towards open models, more and more JuMP/Julia
  - EnergyModelsX
  - TimeStruct
  - SparseVariables

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# Motivation

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- Often need to model nonlinearities by means of **linearization**
- Colleagues or customers often use **more detailed dynamical models** (not *yet* in Julia?)
- Spending computation time on good/optimal approximation may be a good trade-off
- Library with **robust methods**
  - Easy to update with **new data**
  - We hope it can be useful for others
  - We hope to improve the quality and robustness by having more users/contributors

# Overview

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- Generate piecewise affine approximations from point estimates
  - convenience function to sample functions
- Convex/Concave functions
  - method to convexify estimates with numerical errors/noise
- Currently support 3 methods:
  1. MILP to fit a set of points, partially based on Toriello & Vielma, 2012.
  2. Cluster uses a heuristic to fit the set of points, based on Magnani & Boyd, 2009.
  3. Progressive uses a heuristic to add planes until a certain accuracy is met, based on Kazda & Li, 2024.

# Demo

```
1 using JuMP, HiGHS, PlutoUI
```

```
1 using PiecewiseAffineApprox # Now registered in General
```

```
optimizer = OptimizerWithAttributes(HiGHS.Optimizer, [Silent() => true])
```

```
1 optimizer = optimizer_with_attributes(HiGHS.Optimizer, MOI.Silent()=>true)
```

```
x =  
[-1.0, -0.777778, -0.555556, -0.333333, -0.111111, 0.111111, 0.333333, 0.555556, 0.777778,
```

```
1 x = collect(range(-1, 1; length = 10))
```

```
z =  
[1.0, 0.604938, 0.308642, 0.111111, 0.0123457, 0.0123457, 0.111111, 0.308642, 0.604938, 1.0
```

```
1 z = x .^ 2
```

```
pwa = PiecewiseAffineApprox.PWAFunc{PiecewiseAffineApprox.Convex, 1} with 3 planes:  
z ≥ -1.5555555555555555 x1 + -0.5555555555555555  
z ≥ -0.222222222222149085 x1 + 0.037037037037273006  
z ≥ 1.5555555555555556 x1 + -0.5555555555555556
```

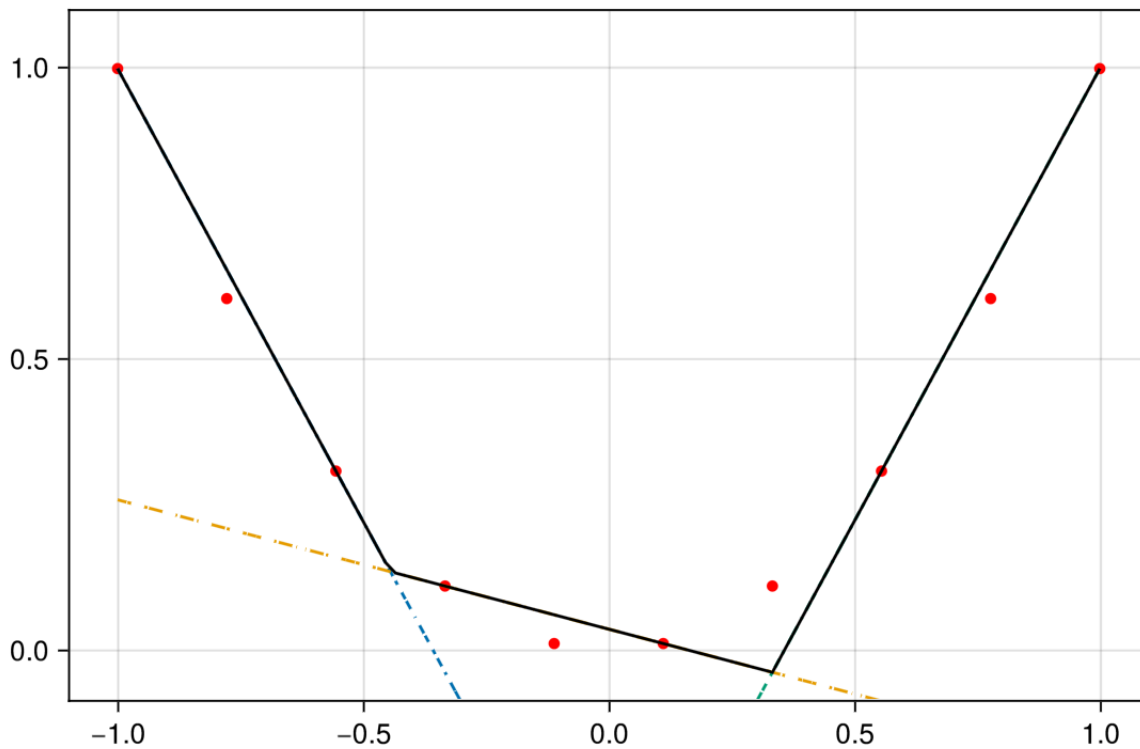
```
1 pwa = approx(  
2     FunctionEvaluations(Tuple.(x), z),  
3     Convex(),  
4     MILP(;optimizer, metric = metric, planes = nplanes, strict=strict),  
5 )
```

```
PWAFunc{Convex, 1}  
planes: Array{Plane{1}}((3,))  
1: Plane{1}  
  α: Tuple{Float64}  
    1: Float64 -1.5555555555555555  
    β: Float64 -0.5555555555555555  
2: Plane{1}  
  α: Tuple{Float64}  
    1: Float64 -0.222222222222149085  
    β: Float64 0.037037037037273006  
3: Plane{1}  
  α: Tuple{Float64}  
    1: Float64 1.5555555555555556  
    β: Float64 -0.5555555555555556
```

```
1 Dump(pwa)
```

## Visualizing the results

```
1 using WGLMakie
```



```
1 plot(x, z, pwa)
```



```
1 @bind nplanes PlutoUI.Slider(1:5; default=3)
```

none ▼

```
1 @bind strict PlutoUI.Select([:none,:inner,:outer]; default=:none)
```

l1 ▼

```
1 @bind metric PlutoUI.Select([:l1,:max,:l2]; default=:l1)
```

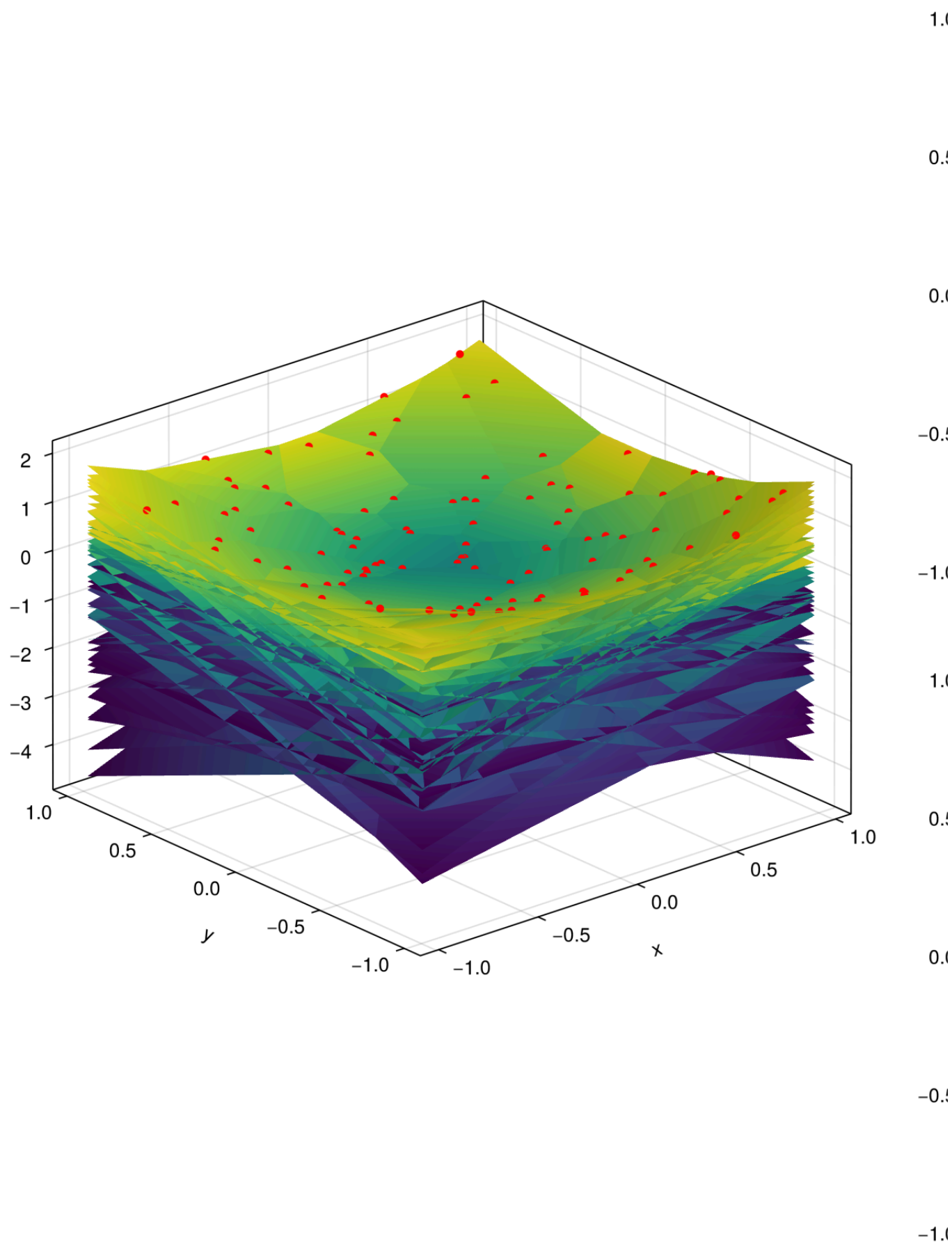
## 3D example

demo (generic function with 1 method)

```
1 function demo()
2     optimizer = optimizer_with_attributes(HiGHS.Optimizer, MOI.Silent()=>true)
3
4     I = 100
5     xmat = 2 * rand(2, I) .- 1
6     x = [Tuple(xmat[:, i]) for i = 1:size(xmat, 2)]
7     z = [p[1]^2 + p[2]^2 for p in x]
8     vals = FunctionEvaluations(x, z)
9
10    pwa = approx(
11        vals,
12        Convex(),
13        Progressive(; optimizer = HiGHS.Optimizer, ),
14    )
15    p = plot(vals, pwa)
16 end
```



$l_1 = 0.12, l_2 = 0.12, \text{max} = 0.12$



demo()

Fitting finished, error = 0.12, p = 100



Running HiGHS 1.7.1 (git hash: 43329e528): Copyright (c) 2024 HiGHS under MIT licence terms  
Coefficient ranges:  
Matrix [6e-04, 1e+00]  
Cost [1e+00, 1e+00]  
Bound [0e+00, 0e+00]  
RHS [7e-04, 2e+00]  
Presolving model  
10100 rows, 10300 cols, 40300 nonzeros 0s  
9746 rows, 9946 cols, 38884 nonzeros 0s  
Presolve : Reductions: rows 9746(-354); columns 9946(-354); elements 38884(-1416)  
Solving the presolved LP  
Using EKK dual simplex solver - serial  
Iteration Objective Infeasibilities num(sum)  
0 -1.1195312825e+02 Ph1: 2209(4490.35); Du: 30(111.953) 0s  
9937 1.0286451127e+04 Pr: 0(0) 0s  
Solving the original LP from the solution after postsolve  
Model status : Optimal  
Simplex iterations: 9937  
Objective value : 1.0286451127e+04  
HiGHS run time : 0.41  
Running HiGHS 1.7.1 (git hash: 43329e528): Copyright (c) 2024 HiGHS under MIT licence terms  
Coefficient ranges:  
Matrix [6e-04, 1e+00]  
Cost [1e-04, 1e+00]  
Bound [0e+00, 0e+00]  
RHS [7e-04, 2e+00]  
Presolving model  
200 rows, 203 cols, 800 nonzeros 0s  
197 rows, 200 cols, 782 nonzeros 0s  
Presolve : Reductions: rows 107(-3); columns 200(-3); elements 782(-18)

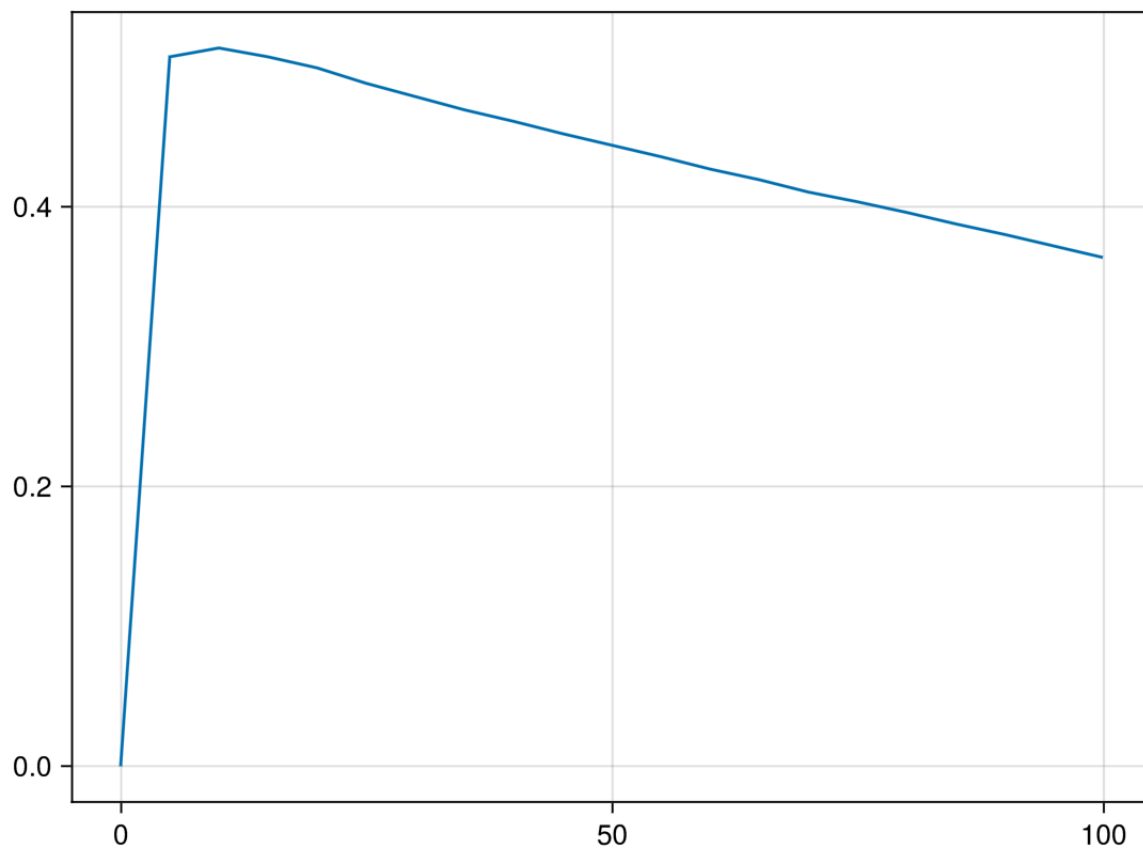
# Energy Applications

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- Process **efficiency**
  - Electrolyser
  - **Fuel Cell**
  - Compression
  - Etc
- When solving repeatedly or with high temporal resolution, **reusing a piecewise convex approximation** pays off in solution time
- ISMP TB870 Example application in the Arctic

# Fuel Cell example

Get system efficiency estimates from dynamic model (Dymola/Modelica):



```
lines(fc_data[:,1],fc_data[:,2])
```

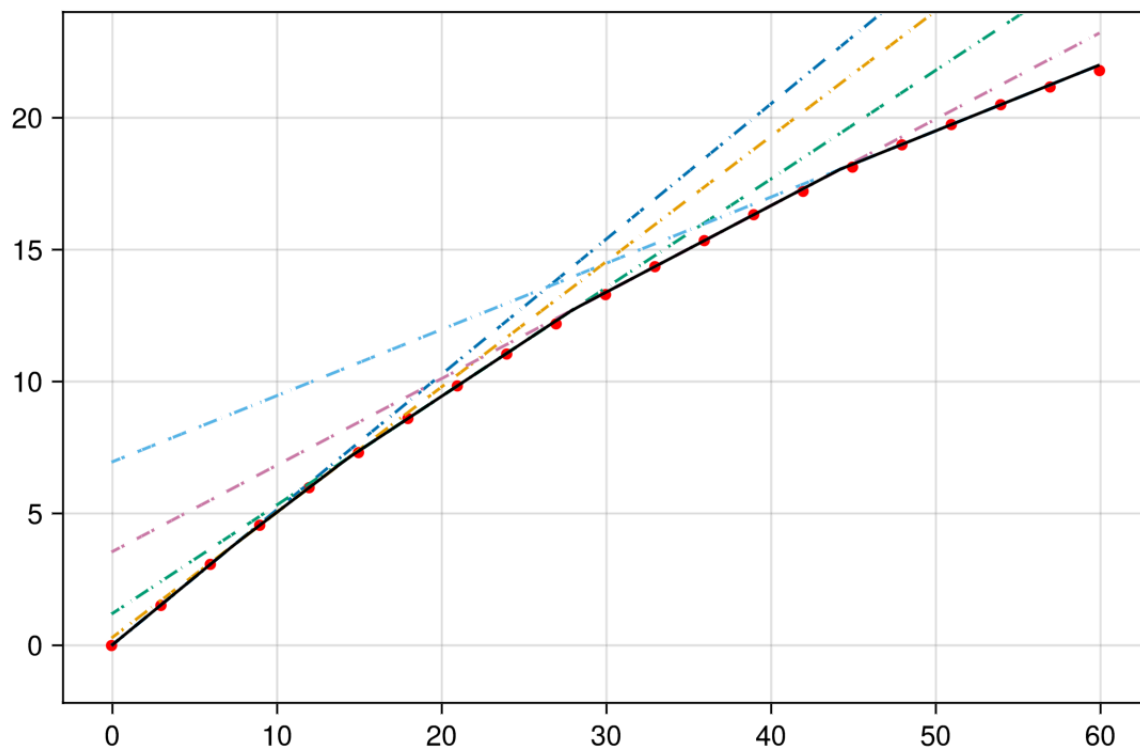
Transform to input->output and linearize:

```
fc_io = transform_io(fc_data, 60);
```

**pwa\_fc** = PiecewiseAffineApprox.PWAFunc{PiecewiseAffineApprox.Concave, 1} with 5 planes:

```
z ≤ -0.5135509999999998 x1 + -0.0  
z ≤ -0.4756269999999997 x1 + -0.284436000000000036  
z ≤ -0.412446000000000064 x1 + -1.19460599999998677  
z ≤ -0.3283910000000000104 x1 + -3.5493119999999961  
z ≤ -0.2512830000000000264 x1 + -6.9621119999999864
```

```
pwa_fc = approx(  
    FunctionEvaluations(Tuple.(fc_io[:,1]), fc_io[:,2]),  
    Concave(),  
    MILP(;optimizer, metric = :l1, planes = 5, strict=:outer),  
)
```



```
plot(fc_io[:,1], fc_io[:,2], pwa_fc)
```

```
transform_io (generic function with 2 methods)
```

```
function transform_io(d, cap=50)
    input = d[:,1] .* 0.01 .* cap
    output = d[:,2] .* input
    io = [input output]
end
```

# Future Improvements

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- Interface improvements
- Improve robustness - better method to calculate  $\text{bigM}$
- More methods
- Give it a try and contribute: <https://github.com/sintefore/PiecewiseAffineApprox.jl>



# Technology for a better society

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